

Black Hole Solutions in String TheoryRamzi R. Khuri[†]*CERN, Theory Division
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ABSTRACT

We present two-parameter solutions of the low-energy four-dimensional heterotic string which in the extremal limit reduce to supersymmetric monopole, string and domain wall solutions. The effective scalar coupling to the Maxwell field, $e^{-\alpha\phi}F_{\mu\nu}F^{\mu\nu}$, gives rise to a new string black hole with $\alpha = \sqrt{3}$, in contrast to the pure dilaton black hole solution which has $\alpha = 1$. Implications of string/fivebrane duality in $D = 10$ to four-dimensional dualities are discussed.

In recent work¹, supersymmetric soliton solutions of the four-dimensional heterotic string were presented, describing monopoles, strings and domain walls. These solutions admit the $D = 10$ interpretation of a fivebrane wrapped around 5, 4 or 3 of the 6 compactified dimensions and are arguably exact to all orders in α' . In this talk, we extend all three solutions to two-parameter solutions of the low-energy equations of the four-dimensional heterotic string². The two-parameter solution extending the supersymmetric monopole corresponds to a magnetically charged black hole, while the solution extending the supersymmetric domain wall corresponds to a black membrane. By contrast, the two-parameter string solution does not possess a finite horizon and corresponds to a naked singularity.

All three solutions involve both the dilaton and the modulus fields, and are thus to be contrasted with pure dilaton solutions³. In particular, the effective scalar coupling to the Maxwell field, $e^{-\alpha\phi}F_{\mu\nu}F^{\mu\nu}$, gives rise to a new string black hole with $\alpha = \sqrt{3}$, in contrast to the pure dilaton solution of the heterotic string which has $\alpha = 1$ ³. It thus resembles the black hole arising from Kaluza-Klein theories which also has $\alpha = \sqrt{3}$, and which reduces to the Pollard-Gross-Perry-Sorkin⁴ magnetic monopole in the extremal limit. The fact that the heterotic string admits $\alpha = \sqrt{3}$ black holes also has implications for string/fivebrane duality⁵. Both electric/magnetic duality and string/string duality in $D = 4$ may be seen as a consequence of string/fivebrane duality in $D = 10$.

We begin with the two-parameter black hole. Inspired by the wrapping of a

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fivebrane around five of the six compactified dimensions $(x_5, x_6, x_7, x_8, x_9)$, it was shown¹ that the tree-level effective action for the $D = 10$ heterotic string may be reduced to the following four-dimensional form

$$S_1 = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} e^{-2\Phi - \sigma_1} \left(R + 4(\partial\Phi)^2 + 4\partial\sigma_1 \cdot \partial\Phi - \frac{1}{4} e^{2\sigma_1} F_{\mu\nu} F^{\mu\nu} \right), \quad (1)$$

where $\mu, \nu = 0, 1, 2, 3$. Here $g_{\mu\nu}$ is the string sigma-model metric and Φ is the dilaton. In the case of toroidal compactification, with $N = 4$ supersymmetry in $D = 4$, σ_1 is a modulus field, $g_{44} = e^{-2\sigma_1}$, and $F_{\mu\nu} = H_{\mu\nu 4}$ where $H = dB$ and B is the string antisymmetric tensor. However, actions of this type also appear in a large class of $N = 1$ supergravity theories⁶. The solution is given by²

$$\begin{aligned} e^{-2\Phi} &= e^{2\sigma_1} = \left(1 - \frac{r_-}{r}\right), \\ ds^2 &= -\left(1 - \frac{r_+}{r}\right) \left(1 - \frac{r_-}{r}\right)^{-1} dt^2 + \left(1 - \frac{r_+}{r}\right)^{-1} dr^2 + r^2 \left(1 - \frac{r_-}{r}\right) d\Omega_2^2, \\ F_{\theta\varphi} &= \sqrt{r_+ r_-} \sin\theta, \end{aligned} \quad (2)$$

where here, and throughout this paper, we set the dilaton vev Φ_0 equal to zero. This represents a magnetically charged black hole with event horizon at $r = r_+$ and inner horizon at $r = r_-$. The magnetic charge and mass of the black hole are given by

$$\begin{aligned} g_1 &= \frac{4\pi}{\sqrt{2}\kappa} (r_+ r_-)^{\frac{1}{2}}, \\ \mathcal{M}_1 &= \frac{2\pi}{\kappa^2} (2r_+ - r_-). \end{aligned} \quad (3)$$

Changing coordinates via $y = r - r_-$ and taking the extremal limit $r_+ = r_-$ yields:

$$\begin{aligned} e^{2\Phi} &= e^{-2\sigma_1} = \left(1 + \frac{r_-}{y}\right), \\ ds^2 &= -dt^2 + e^{2\Phi} (dy^2 + y^2 d\Omega_2^2), \\ F_{\theta\varphi} &= r_- \sin\theta, \end{aligned} \quad (4)$$

which is just the tree-level supersymmetric monopole solution without a Yang-Mills field which saturates the Bogomol'nyi bound $\sqrt{2}\kappa\mathcal{M}_1 \geq g_1$. Note that the monopole arises in the gravitational sector of the string, as can be seen from an earlier solution found in purely bosonic string theory⁷. Supersymmetric extensions of this solution both with and without gauge fields were found in⁸.

Next we derive a two-parameter string solution which, however, does not possess a finite event horizon and consequently cannot be interpreted as a black string. This is inspired by the wrapping of the fivebrane around four of the compactified dimensions (x_6, x_7, x_8, x_9) . The action is given by

$$S_2 = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} e^{-2\Phi - 2\sigma_2} \left(R + 4(\partial\Phi)^2 + 8\partial\sigma_2 \cdot \partial\Phi + 2(\partial\sigma_2)^2 - \frac{1}{2} e^{4\sigma_2} F_\mu F^\mu \right), \quad (5)$$

In the case of the torus, σ_2 is the modulus field $g_{44} = g_{55} = e^{-2\sigma_2}$ and $F_\mu = H_{\mu 45}$. A two-parameter family of solutions is now given by²

$$\begin{aligned} e^{2\Phi} &= e^{-2\sigma_2} = (1 + k/2 - \lambda \ln y), \\ ds^2 &= -(1 + k)dt^2 + (1 + k)^{-1}(1 + k/2 - \lambda \ln y)dy^2 + y^2(1 + k/2 - \lambda \ln y)d\theta^2 + dx_3^2, \\ F_\theta &= \lambda\sqrt{1 + k}, \end{aligned} \tag{6}$$

where for $k = 0$ we recover the supersymmetric string soliton solution¹ which is dual to the elementary string solution of Dabholkar *et al*⁹. The solution shown in Eq.(6) in fact represents a naked singularity, since the event horizon is pushed out to $r_+ = \infty$, which agrees with the Horowitz-Strominger “no-4D-black-string” theorem¹⁰.

Finally, we consider the two-parameter black membrane solution. In this case, we wrap the fivebrane around three of the compactified dimensions (x_7, x_8, x_9) . However, the four-dimensional action necessary to yield membrane solutions is not obtained by a simple dimensional reduction of the ten-dimensional action because of the non-vanishing of $F = H_{456}$. Instead, the effective action is obtained by treating F^2 as a cosmological constant and is given by

$$S_3 = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} e^{-2\Phi - 3\sigma_3} \left(R + 4(\partial\Phi)^2 + 12\partial\sigma_3 \cdot \partial\Phi + 6(\partial\sigma_3)^2 - e^{6\sigma_3} \frac{1}{2} F^2 \right), \tag{7}$$

In the case of the torus, σ_3 is the modulus field $g_{44} = g_{55} = g_{66} = e^{-2\sigma_3}$. The two-parameter black membrane solution is then²

$$\begin{aligned} e^{-2\Phi} &= e^{2\sigma_3} = \left(1 - \frac{r}{r_-}\right), \\ ds^2 &= -\left(1 - \frac{r}{r_+}\right) \left(1 - \frac{r}{r_-}\right)^{-1} dt^2 + \left(1 - \frac{r}{r_+}\right)^{-1} \left(1 - \frac{r}{r_-}\right)^{-4} dr^2 + dx_2^2 + dx_3^2, \\ F &= -(r_+ r_-)^{-1/2}. \end{aligned} \tag{8}$$

This solution represents a black membrane with event horizon at $r = r_+$ and inner horizon at $r = r_-$. Changing coordinates via $y^{-1} = r^{-1} - r_-^{-1}$ and taking the extremal limit yields

$$\begin{aligned} e^{2\Phi} &= e^{-2\sigma_3} = \left(1 + \frac{y}{r_-}\right), \\ ds^2 &= -dt^2 + dx_2^2 + dx_3^2 + e^{2\Phi} dy^2, \\ F &= -\frac{1}{r_-}. \end{aligned} \tag{9}$$

which is just the supersymmetric domain wall solution¹.

Consider the generic toroidal compactification of the heterotic string, where the four-dimensional theory is given by $N = 4$ supergravity coupled to 22 $N = 4$ vector multiplets. Then the Maxwell field $F_{\mu\nu}$ in $D = 4$ (or its dual $\tilde{F}_{\mu\nu}$) and the scalar field ϕ come from the $D = 10$ 3-form (or 7-form) and dilaton plus modulus field of the heterotic string (or heterotic fivebrane). Thus, the $D = 4$ electric/magnetic duality can be interpreted as a reduction of $D = 10$ string/fivebrane duality.

Another reduction of string/fivebrane duality is $D = 4$ string/string duality¹. The compactified heterotic string displays a target space duality $O(6, 22, Z)$. It is also conjectured to display the strong/weak coupling $SL(2, Z)$ S -duality relating the dilaton and the axion, which is certainly there in the field theory limit. The “duality of dualities” suggestion^{11,1} is that, under string/fivebrane duality, the roles of S and T dualities are interchanged. The picture that emerges is one in which the massive states of the string correspond to extreme black holes.

We have shown only that these two-parameter configurations are solutions of the field theory limit of the heterotic string. Although the extreme one-parameter solutions are expected to be exact to all orders in α' , the same reasoning does not carry over to the new two-parameter solutions. It would be also interesting to see whether the generalization of the one-parameter solutions to the two-parameter solutions can be carried out when we include the Yang-Mills coupling. This would necessarily involve giving up the self-duality condition on the Yang-Mills field strength, however, since the self-duality condition is tied to the extreme, $\sqrt{2}\kappa\mathcal{M}_{p+1} = g_{p+1}$, supersymmetric solutions. Finally, there is the question of whether these solutions are peculiar to the toroidal compactification or whether they survive in more realistic orbifold or Calabi-Yau models. Although the actions S_1 , S_2 and S_3 were originally derived in the context of the torus¹, they also appear in a large class of $N = 1$ supergravity theories.

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